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Ultrafast Optics

All Exercises

Winter term 2020/2021

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1. Gaussian beam focusing

a) *A mode-locked Ti:sapphire oscillator emits Gaussian pulses with a pulse length of* $\tau = 50\,\mathrm{fs}$ *at a carrier wavelength of* $\lambda_0 = 800\,\mathrm{nm}$, *carrying an energy of* $E = 50\,\mathrm{nJ}$. *The beam diameter is* $D = 5\,\mathrm{mm}$. *Calculate the peak intensity and the corresponding electric field amplitude occurring in the focus of a lens with the focal length* $f = 10\,\mathrm{mm}$. *(Hint: Please assume* D *and* τ *to be defined as FWHM of the beam/pulse).*

b) *Which power must be reached in order to overcome the field strength* ($\geq 10^{10}\,\frac{\mathrm{V}}{\mathrm{cm}}$) *between the nucleus and electrons of an atom (pulsed Nd:YAG laser, spot diameter* $5\,\mu\mathrm{m}$*)? A typical Nd:YAG laser is operating at* $\lambda = 1064\,\mathrm{nm}$ *and emits pulses with a pulse duration of around 10 ns.*

a.) Solution: The intensity of the short laser pulse can be written as a function of the propagation direction z and the transversal coordinate r

$$I(r,z) = I_0\left(\frac{W_0'}{W'(z)}\right)^2 \exp\left(-\eta\frac{r^2}{(2W_0')^2}\right) \overset{z=0}{=} I_0 \exp\left(-\eta\frac{r^2}{(2W_0')^2}\right). \tag{1.1}$$

For different definition of the pulse with we introduced the parameter η which varies for different criteria

$$\eta = \begin{cases} 4\ln 2 & \text{FWHM} \\ 4 & \frac{1}{e} \\ 8 & \frac{1}{e^2} \end{cases}. \tag{1.2}$$

We can now calculate the power of the beam by integrating the intensity over the whole x-y-plane which leads to

$$P = \int_0^{2\pi}\int_0^{\infty} I(r)\,r\,\mathrm{d}r\,\mathrm{d}\varphi = 2\pi I_0 \int_0^{\infty} r\exp\left(-\eta\frac{r^2}{(2W_0')^2}\right)\mathrm{d}r = \frac{\pi}{\eta}(2w_0')^2 I_0. \tag{1.3}$$

This leads to an expression of the Intensity for a given power P

$$I_0 = \frac{P\eta}{\pi(2w_0')^2}. \tag{1.4}$$

We can now calculate the whole pulse energy by integrating over time. We assume a Gaussian shaped envelope

$$P(E,t) = P_0(E)\exp\left(-\eta\frac{t^2}{\tau^2}\right), \tag{1.5}$$

which leads to the pulse energy

$$E = \int_{-\infty}^{\infty} P(E,t)\,\mathrm{d}t = P_0\tau\frac{\pi}{\eta} \quad \Rightarrow P_0(E) = \sqrt{\frac{\eta}{\pi}}\frac{E}{\tau}. \tag{1.6}$$

For a lens of focal length f the diameter of the Gaussian beam at the focal point of the lens is

$$w_0' = \frac{f \cdot \lambda}{\pi w_L}, \quad 2w = D. \tag{1.7}$$

We can now use (1.4) to compute the intensity

$$I_0 = \frac{P_0 \eta}{\pi (2w_0')^2} = \frac{\eta}{4\pi}\sqrt{\frac{\eta}{\pi}}\frac{E}{\tau}\frac{\pi^2 w_L^2}{f^2 \lambda^2} = \frac{1}{4}\sqrt{\pi \eta^3}\left(\frac{w_L^2}{f\lambda}\right)\frac{E}{\tau} = 2 \cdot 10^{17}\,\frac{\mathrm{W}}{\mathrm{m}^2}. \tag{1.8}$$

We can now deduce the electric field strength by using

$$I_0 = \frac{c\varepsilon_0 E^2}{2} \quad \Rightarrow E = \sqrt{\frac{2I_0}{c\varepsilon_0}} = 1{,}23\,\frac{\mathrm{V}}{\mathrm{m}}. \tag{1.9}$$

b.) Solution: We use the former relation between the intensity and the electric field to find a formula for the puls peak power

$$E = \sqrt{\frac{2I_0}{c\varepsilon_0}} = \sqrt{\frac{2\eta P}{\pi (2w_0)^2 c\varepsilon_0}} \geq 10^{12}\,\frac{\mathrm{V}}{\mathrm{m}} = \tilde{E}. \tag{1.10}$$

This leads to

$$P \geq \frac{\pi (2w_0)^2 c\varepsilon_0 \tilde{E}^2}{2\eta} = 3{,}76 \cdot 10^{10}\,\mathrm{W}. \tag{1.11}$$

The corresponding pulse energy is

$$E = P_0 \tau \sqrt{\frac{\pi}{\eta}} = 400\,\mathrm{J}. \tag{1.12}$$

2. Pulse train characteristics

a) *Derive and calculate the spectrum of the Ti:Sa laser system from above assuming a repetition rate of* $f_{rep} = 80\,\mathrm{MHz}$.

b) *How would the measured spectrum look if the switch was set to transmit only a single pulse?*

Solution In order to derive the spectrum, we assume an infinite pulse train of Gaussian shaped pulses with a temporal distance of $\xi = 1/f_{\mathrm{rep}}$. We calculate the pulse train as a convolution of the Gaussian pulses with an infinite comb of Dirac-Deltas with a sampling distance ξ. We can therefore write

$$f(t) = \exp\left(-\eta\frac{t^2}{\tau^2}\right), \qquad \sum_{n=-\infty}^{\infty}\delta(t-\xi n). \tag{2.13}$$

We can write the convolution as

$$\begin{aligned} h(t) = f(t) * g(t) &= \int_{-\infty}^{\infty} f(t-t')g(t')\,\mathrm{d}t' = \int_{-\infty}^{\infty}\left[f(t-t')\sum_{n=-\infty}^{\infty}\delta(t'-\xi n)\right]\mathrm{d}t' \\ &= \sum_{n=-\infty}^{\infty} f(t-\xi n) = \sum_{n=-\infty}^{\infty}\exp\left(-\eta\frac{(t-\xi n)^2}{\tau^2}\right). \end{aligned} \tag{2.14}$$

However, it is much more convenient to use the property of the Fourier transfrom

$$FT[h(t)] = FT[f(t) * g(t)] = FT[f(t)]\cdot FT[g(t)] \tag{2.15}$$

and compute both Fourier transforms separately

$$FT[f(t)] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp\left(-\eta\frac{t^2}{\tau^2}\right)\exp(-\mathrm{i}\omega t)\,\mathrm{d}t = \int_{-\infty}^{\infty}\exp\left(-\frac{\eta}{\tau^2}t^2 - \mathrm{i}\omega t\right)\mathrm{d}t. \tag{2.16}$$

We can use the Gaussian integral $\int_{-\infty}^{\infty}\exp\left(-ax^2+bx\right)\mathrm{d}x = \sqrt{\frac{\pi}{a}}\exp\left(\frac{b^2}{4a}\right)$,

$$\Rightarrow FT[f(t)] = \frac{1}{\sqrt{2\pi}}\frac{\pi}{\eta}\tau\exp\left(-\frac{\omega^2}{4\eta}\tau^2\right) = \tau\sqrt{2\eta}\exp\left(-\frac{\omega^2}{4\eta}\tau^2\right). \tag{2.17}$$

The comb of Delta Functions can be Fourier transformed as follows

$$\begin{aligned} FT[g(t)] &= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\delta(t-\xi n)\exp(-\mathrm{i}\omega t)\,\mathrm{d}t = \frac{1}{\sqrt{2\pi}}\sum_{n=-\infty}^{\infty}\exp(-\mathrm{i}\omega\xi n) \\ &\text{use identity: } \sum_{n=-\infty}^{\infty}P\delta(x-nP) = \sum_{n=-\infty}^{\infty}\exp\left(-2\mathrm{i}\pi\frac{n}{P}x\right) \\ &= \frac{1}{\sqrt{2\pi}}\sum_{n=-\infty}^{\infty}\exp\left(-2\mathrm{i}\pi\frac{\xi n}{2\pi}\omega\right) = \frac{\sqrt{2\pi}}{\xi}\sum_{n=-\infty}^{\infty}\delta\left(\omega-\frac{2\pi n}{\xi}\right). \end{aligned} \tag{2.18}$$

We can now put (2.17) and (2.18) together

$$\begin{aligned} FT[h(t)] &= \frac{\tau}{\sqrt{2\eta}} \exp\left(-\frac{\tau^2}{4\eta}\omega^2\right) \frac{\sqrt{2\pi}}{\xi} \sum_{n=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi m}{\xi}\right) \\ &\propto \exp\left(-\frac{\tau^2}{4\eta}\omega^2\right) \sum_{n=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi n}{\xi}\right). \end{aligned} \tag{2.19}$$

The spectrum therefore also consists of delta functions, which are modulated by a Gaussian envelope, which FWHM is proportional to $1/\tau^2$. This shows, that shorter pulses have a broader spectrum. For a single pulse $FT[f(t)]$ the spectrum is also a Gaussian function, whereas for a pulse train the spectrum is not continuous but consists of delta functions, which are spaced by $2\pi/\xi$.

Generally we can identify the parameter ξ as the round trip time T in the resonator

$$T = \xi = \frac{2l}{c} = \frac{1}{f_{\text{rep}}}. \tag{2.20}$$

For a repition rate $f_{\text{rep}} = 80\,\text{MHz}$ the frequency spacing can be calculated as

$$\Delta\omega = \frac{2\pi}{\xi} = \frac{2\pi}{T} \quad \Rightarrow \Delta\nu = \frac{1}{T} = f_{\text{rep}} = 80\,\text{MHz}. \tag{2.21}$$

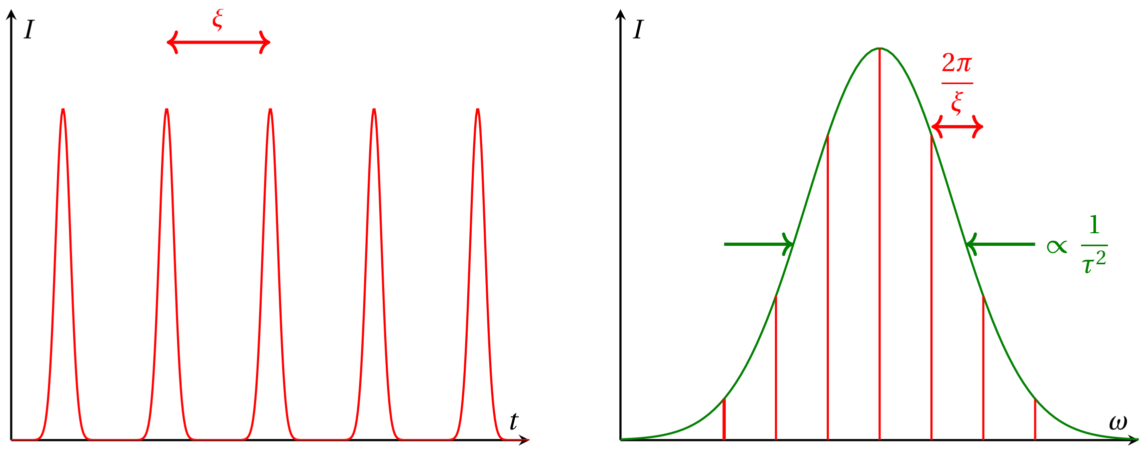


Fig. 1: Left: temporal intensity of the pulse train consisting of thin Gaussian beams.
Right: Calculated spectrum, which is made of delta functions, which are modulated by a Gaussian envelope.

3 Spectrometer and Resonator characteristics

a) *Sketch and compare a prism spectrometer and a grating spectrometer. Which parameters influence their respective resolutions $\lambda / \mathrm{d}\lambda$?*

b) *A prism with $a = 3\,\mathrm{cm}$ (base length) composed of flint glass ($n(480\,\mathrm{nm}) = 1.8297$ and $n(546{,}6\,\mathrm{nm})$) is used. Calculate the resolution.*

c) *Is it possible to use a grating spectrometer in order to resolve the longitudinal modes of the laser described in the previous task? Substantiate your answer mathematically. Do you know other techniques to resolve longitudinal modes?*

d) *What is the maximum length of a He-Ne laser (spectral line width $\Delta f_l = 1{,}5\,\mathrm{GHz}$ and CO_2 laser ($\Delta f_l = 60\,\mathrm{MHz}$) to oscillate only with 1 longitudinal mode?)*

e) *Estimate the minimal achievable pulse duration of a Ti:Sa laser with the given emission spectra 2 for* sech^2 *pulses.*

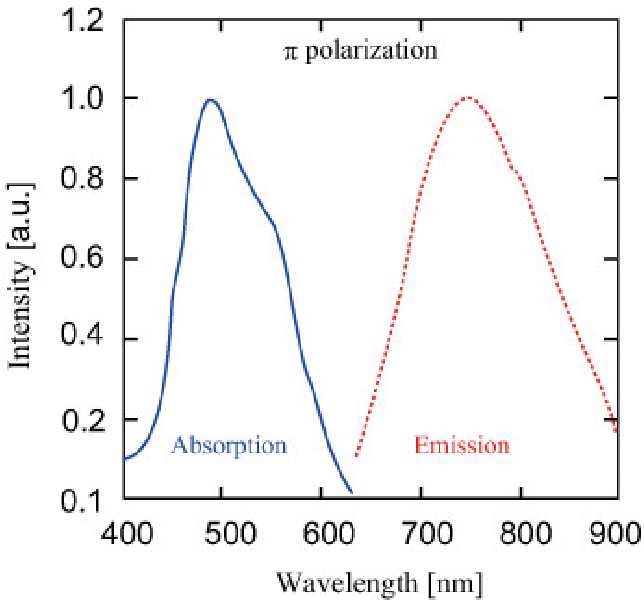
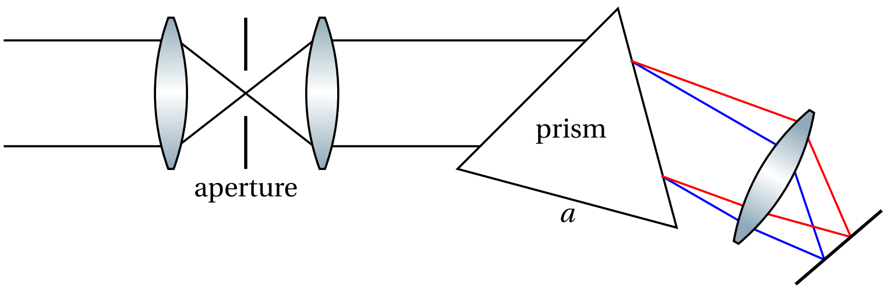


Fig. 2: Absorption and emission spectra of a Ti:Sa crystal.

a.) Solution:

At first we discuss the prism spectrometer:

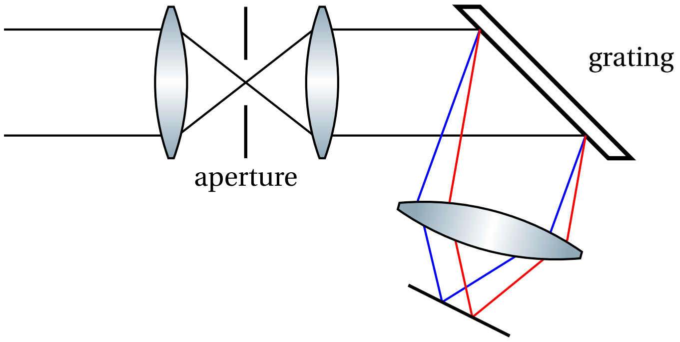


The resolution of a prism spectrometer is given as

$$\frac{\lambda}{\mathrm{d}\lambda} = a \cdot \left|\frac{\mathrm{d}n}{\mathrm{d}\lambda}\right|, \tag{3.1}$$

where a is the prism base length. The refractive index n of the material is a function of the wavelength and causes a dispersion $\frac{\mathrm{d}n}{\mathrm{d}\lambda}$.

We can now discuss the grating spectrometer: The resolution of the spectrometer is deter-



mined by the number of illuminated grating lines N which leads to

$$\frac{\lambda}{\mathrm{d}\lambda} = m \cdot N, \tag{3.2}$$

where m is the order of diffraction.
Now we want to compare the advantages and disadvantages of both spectrometer types:

Table 1: Comparison of the grating and prism spectrometer

	Grating spectrometer		Prism spectrometer
+	in most cases higher resolution	–	not as high resolution
–	limited free spectral range due to the overlap of high order diffraction. However, the spectral range can be increased by adding color filters	+	free spectral range not limited
–	polarization dependent	–	prism material limits wavelength range
–	wavelength dependent diffraction efficiency	+	higher output intensity as there is only one *order*
		+	polarization independent
		–	lower damage threshhold (issue for pulsed lasers)

b.) Solution: We can calculate the resolution of the prism spectrometer by using equation (3.1)

$$\frac{\lambda}{\mathrm{d}\lambda} = a \cdot \left|\frac{\mathrm{d}n}{\mathrm{d}\lambda}\right| \approx 0{,}03\,\mathrm{m} \cdot \left|\frac{1.8126 - 1.8297}{(546.6 - 480) \cdot 10^{-9}\,\mathrm{m}}\right| = 7.7 \cdot 10^{3}. \tag{3.3}$$

This leads to a resolution at 480 nm of

$$\Delta\lambda = \frac{\lambda}{7.7 \cdot 10^3} \approx 50 \cdot 10^{-3}\,\text{nm} = 5\,\text{pm}. \tag{3.4}$$

c.) Solution: A Ti:Sa laser system works in a wavelength regime of $\lambda = 800\,\text{nm}$. This leads to a laser frequency of $\nu = 3{,}75 \cdot 10^{14}\,\text{Hz}$. The mode spacing is determined by the repetition rate $\Delta\nu = f_{\text{rep}} = 80\,\text{MHz}$.

For a typical beam diameter $D = 5\,\text{mm}$ and assuming we use the first order diffraction of the grating ($m = 1$) we can calculate the required grating period Λ in order to resolve the longitudinal modes

$$\frac{\lambda}{\Delta\lambda} = m \cdot N = m\frac{D}{\Lambda}. \tag{3.5}$$

For $\frac{\Delta\lambda}{\lambda} \approx \frac{\Delta\nu}{\nu}$ we find

$$\begin{aligned}\Lambda &= \frac{\Delta\lambda}{\lambda} m \cdot D = \frac{\Delta\nu}{\nu} m \cdot D \\ &= \frac{80\,\text{MHz}}{3{,}75 \cdot 10^{14}\,\text{Hz}} \cdot 5\,\text{mm} = 1\,\text{nm} \ll 800\,\text{nm}.\end{aligned} \tag{3.6}$$

Because the required grating period is much smaller than the wavelength λ of the laser, we get no refraction. The smallest possible period is determined by the abbe limit $\Lambda = \frac{\lambda}{2}$ which is much larger than what we need. The solution to the problem to use a Fabry-Perot interferometer instead.

d.) Solution: In order to achieve single-mode lasing, we need to ensure that only one resonator mode lies inside the spectral line width Δf_l. Therefore the free spectral range (mode spacing) must be larger or equal to the free spectral range

$$\Delta\nu = \frac{c}{2L} \stackrel{!}{=} \Delta f_l \quad \Rightarrow \quad L = \frac{c}{2\Delta f_l}. \tag{3.7}$$

For the two lasers we obtain the following lengths:

$$L = \frac{c}{2\Delta f_l} = \begin{cases} 10\,\text{cm} & \text{He-Ne laser} \\ 2{,}5\,\text{m} & \text{CO}_2\text{ laser}\end{cases} \tag{3.8}$$

e.) Solution: We can calculate the minimal pulse duration by using the bandwidth product

$$\delta t \cdot \delta\nu = K, \qquad K = \begin{cases} 0.315 & \text{sech} \\ 0.441 & \text{Gaussian}\end{cases}. \tag{3.9}$$

From figure 2 we can read of the bandwidth $\delta\lambda = 160\,\text{nm}$. We can use the approximation

$$\frac{\delta\nu}{\nu} = \frac{\delta\lambda}{\lambda} \quad \Rightarrow \quad \delta\nu = \frac{\delta\lambda}{\lambda}\nu = \frac{c}{\lambda^2}\delta\lambda. \tag{3.10}$$

With a centre wavelength of 750 nm we find for the bandwidth

$$\delta\nu = \frac{c}{(750\,\mathrm{nm})^2} 160\,\mathrm{nm} = 85{,}27\,\mathrm{THz}. \tag{3.11}$$

Now we can calculate the minimal pulse duration as

$$\delta t = \frac{0.315}{85{,}27\,\mathrm{THz}} = 3{,}7\,\mathrm{fs}. \tag{3.12}$$

4 Generation of ultra-short pulses

a) *Explain the principle of Kerr-lens mode locking and sketch a corresponding cavity. What are the main differences to Q-switching?*

b) *Estimate the minimal pulse duration of two Q-switched lasers with a corresponding cavity length of 1 m and 0,3 m, respectively. How can this be achieved? How does the pulse duration change if it takes* $2, 3, \dots$ *round trips to dump the total energy stored in the cavity?*

c) *What are the main differences of an active Q switched compared to a passive Q switched laser? Describe solutions to shorten the pulse duration of Q-switched pulses.*

d) *Calculate the achievable pulse length of a mode-locked Argon laser (resonator length* $l = 1\,\mathrm{m}, \lambda = 488\,\mathrm{nm}$, *line width* $\Delta f_l = 4\,\mathrm{GHz}$*).*

e) *Estimate the amount of modes and pulse duration in a 1 m cavity for a Argon laser (gain spectra* $\approx 7\,\mathrm{GHz}$*), dye laser (gain spectra* $\approx 10\,\mathrm{THz}$*) and Ti:Sa laser.*

a.) Solution: Kerr-lens mode locking is based on the nonlinear Kerr-effect, which describes the intensity dependent refractive index in a 3rd order nonlinear medium. Since the pulse has a transverse intensity profile, the refractive index will take a similar shape. This leads to a gradient of refractive index in transverse direction, which acts as a lens.

There are two different ways to achieve mode locking with the Kerr-effect. The first is *hard aperture* mode locking. Here we insert a simple aperture inside the cavity which blocks light which is not on the optical axis. If the resonator produces laser pulses, the intensity is higher and the pulse is focused in the Kerr-medium which leads to a smaller beam diameter. Therefore the laser pulses are preferred to the CW modes. The second way is to use *soft aperture* mode locking. Here we design the cavity in such a way that the beam overlaps only with the active medium when we have a pulse inside the cavity. This can be achieved by using a small active medium. Here, the CW beam only overlaps partially with the active medium and is suppressed.

mode locking	Q switching
repetition time equal to round trip time	repetition time larger than round trip time
pulse travels inside the cavity	inversion builds up, then the energy is suddenly extracted
emitted pulse are copies of original pulse	no correlation between pulses
no temporal jitter	high temporal jitter (passive Q switching)
periodic modulation	step like modulation
recovery time faster than RTT	recovery time faster than decay time of the upper laser level
pulse forms gradually over several hundred round trips	pulse formation follows modulation

b.) Solution: The minimal pulse duration can be determined by assuming that the energy is extracted in one round trip

$$\begin{aligned} T = \frac{2L}{c} &= \frac{2\,\mathrm{m}}{3\cdot 10^8\,\frac{\mathrm{m}}{\mathrm{s}}} = 6{,}66\,\mathrm{ns} \quad (1\,\mathrm{m}) \\ &= \frac{0{,}3\,\mathrm{m}}{3\cdot 10^8\,\frac{\mathrm{m}}{\mathrm{s}}} = 1\,\mathrm{ns} \quad (0{,}3\,\mathrm{m}). \end{aligned} \tag{4.1}$$

We can achieve this pulse duration by dumping the energy inside the cavity in a single round trip. We can use an acousto- or electro-optical modulator, which couples the pulse out of the resonator. If it takes several round trips to dump the total energy, then the pulse duration will increase.

c.) Solution

active Q switching	passive Q switching
external modulation	modulation driven by the pulse itself
higher τ	lower τ
fixed pulse frequency	pulse frequency can not be controlled, leads to temporal jitter

We can shorten the pulse duration of a Q-switched laser by reducing the cavity length and include cavity dumping where we reduce τ_{Ph} by gating the out-coupling. Another method is to increase the losses in the low-Q-state which leads to a build-up of higher inversion and thus reducing the pulse duration according to this formula:

$$\tau = \tau_{\mathrm{Ph}} \frac{n_i - n_f}{n_i - n_{\mathrm{th}}(1 + \ln \frac{n_i}{n_{\mathrm{th}}})}. \tag{4.2}$$

We see that by increasing $n_i - n_{\mathrm{th}}$ and decreasing τ_{Ph}, the pulse duration reduces.

d.) Solution: For the mode-locked Argon lase the achievable pulse length is determined by the bandwidth product

$$\delta t \geq \frac{0.441}{4\,\mathrm{GHz}} 100\,\mathrm{ps} \tag{4.3}$$

e.) Solution: We can estimate the amount of modes by calculating

$$N = \frac{\Delta\nu}{\delta\nu}, \qquad \delta\nu = \frac{c}{2L} = 150\,\mathrm{MHz}. \tag{4.4}$$

The pulse duration can again be obtained by the time-bandwidth product

$$\delta t = \frac{0.441}{\Delta\nu} = \begin{cases} 63\,\mathrm{ps} & \text{Argon} \\ 44\,\mathrm{fs} & \text{Dye} \\ 4{,}7\,\mathrm{fs} & \text{Ti:Sa} \end{cases}, \qquad N = \frac{\Delta\nu}{150\,\mathrm{MHz}} = \begin{cases} 47 & \text{Argon} \\ 6.7\cdot 10^4 & \text{Dye} \\ 6.2\cdot 10^5 & \text{Ti:Sa} \end{cases}. \tag{4.5}$$

5 Dispersion and Compression

a) *Sketch and explain two different ways to compensate the group velocity dispersion (GVD) introduced by a medium exhibiting normal dispersion. Why is GVD compensation important for the generation of ultrashort pulses?*

b) *Derive the GVD for a pair of parallel gratings.*

c) *For what purpose could it be necessary to induce additional GVD?*

d) *Explain the advantages of utilizing gratings instead of prisms in a pulse compressor.*

a.) Solution: There are many different methods to compensate group velocity dispersion. One possibility is to use a setup of parallel gratings. However, two gratings will lead to a spatial separation of the colours which can be compensated by either a mirror or two more gratings. We can also compensate second order dispersion by using the angular dispersion of prisms. We can adjust the GDD by manipulating the distance between both prisms. The spatial separation can be compensated by two extra prisms or a mirror. A third method is to use chirped mirrors which consists of different sized layers reflecting the spectral components in different depths. GVD compensation is especially important because it causes pulse broadening.

b.) Solution: The optical path length in a setup of two graings is

$$S_{\text{opt.}} = \frac{L}{\sin\vartheta} + L\sin\gamma = \frac{L}{\sin\vartheta}(1+\sin\vartheta\sin\gamma), \tag{5.1}$$

when γ and ϑ are the incoming and outgoing angles of the beam with respect to the grating surface. The accumulated pphase is

$$\phi(\omega) = \frac{\nu}{c} S_{\text{opt.}}(\omega). \tag{5.2}$$

Using the grating equation $\cos\vartheta - \cos\gamma = \frac{\lambda}{d}$ we find that the second derivative is

$$\left.\frac{\mathrm{d}^2\phi}{\mathrm{d}\omega^2}\right|_{\omega_0} = -\frac{\lambda^2 L}{2\pi c^2 d^2 \cos^2\vartheta} \quad \Rightarrow \quad \mathrm{GVD} = \frac{\mathrm{GDD}}{L} = -\frac{\lambda^2}{2\pi c^2 d^2 \cos^2\vartheta}. \tag{5.3}$$

c.) Solution: One use of additional GVD is the stretching of pulse e. g. in chirped pulse amplification (CPA). It is also utilized when the bandwidth of the pulse is increased by *self phase modulation* in order to decrease the bandwidth limited pulse duration. In a third order nonlinear medium additional chirp is imprinted onto the spectrum which can be used to compress the pulse to smaller pulse durations.

d.) Solution: Generally gratings have no material dispersion and can compress a pulse more effectively because they inflict more dispersion. They also have a very high damage threshold which is important in pulse compressors. It is also easier to match the stretching and compressing in a CPA system by using gratings.

6 Autocorrelation

a) *Sketch the setup of a background-free SHG autocorrelator and explain its principle of operation. What information can be gained from measurements with that setup and what are its limitations?*

b) *Compare the measured signal obtained by a SHG autocorrelator in the interferometric (with background) and background-free case. How does the autocorrelation trace looks like from a chirped and unchirped pulse?*

c) *Figure out the autocorrelations of a single rectangular pulse (temporal width τ), a single Gaussian pulse and a pulse comprising three identical equidistant Gaussian peaks separated by twice the pulse duration τ (FWHM). Specify characteristic dimensions of the autocorrelation functions and their relation to the parameters of the respective pulses.*

a.) Solution: The autocorrelator is sketched in figure 3. The basic idea is to split the original pulse into two replicas with a time delay and focus them into a nonlinear crystal which generates the Second harmonic. Now we can measure the spectrum as a function of the delay τ and find the autocorrelation function. With the measurements we can gain information

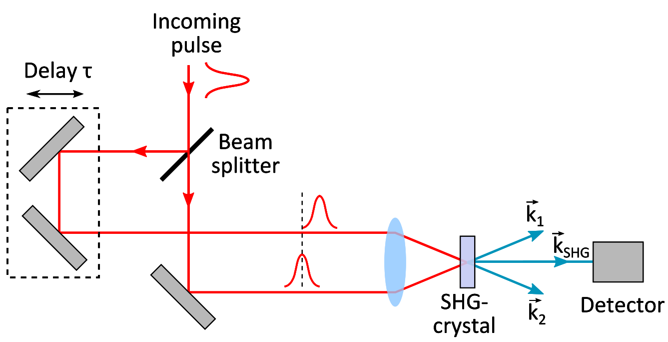


Fig. 3: Setup for background free intensity autocorrelation.

about the pulse duration and approximate shape. However, the signal is symmetric in time, therefore we cannot retrieve the actual pulse shape. Furthermore it requires a stable pulse shape and we gain no phase information from the signal.

b.) Solution: In the background-free case the background term (optical rectification) of the SH is suppressed. This is achieved by overlapping both replicas under an angle in the nonlinear crystal. However, by doing this we lose possible information about second order dispersion which depend on the spectral phase.

Comparing the autocorrelation traces of a chirped and unchirped pulse, one finds that the chirped signal is a little bit broader than the unchirped signal, but still has the same shape. Higher order dispersion mostly manifest themselves in the wings of the autocorrelation function.

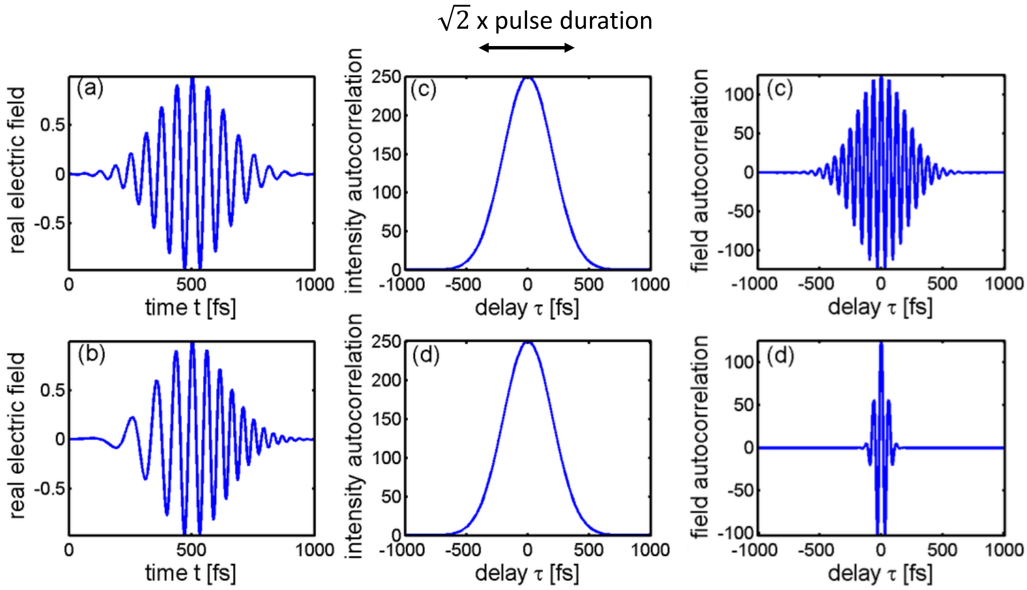


Fig. 4: Middle: Background free autocorrelation function of a bandwidth limited pulse (top) and chirped pulse (bottom).
Right: Interferometric autocorrelation function.

c.) Solution: The autocorrelation signal of a rectangular shaped pulse with temporal width τ is a triangular shaped signal with a baseline 2τ. The autocorrelation signal of a single Gaussian pulse is also Gaussian shaped with a new width of $\sqrt{2}\tau$.

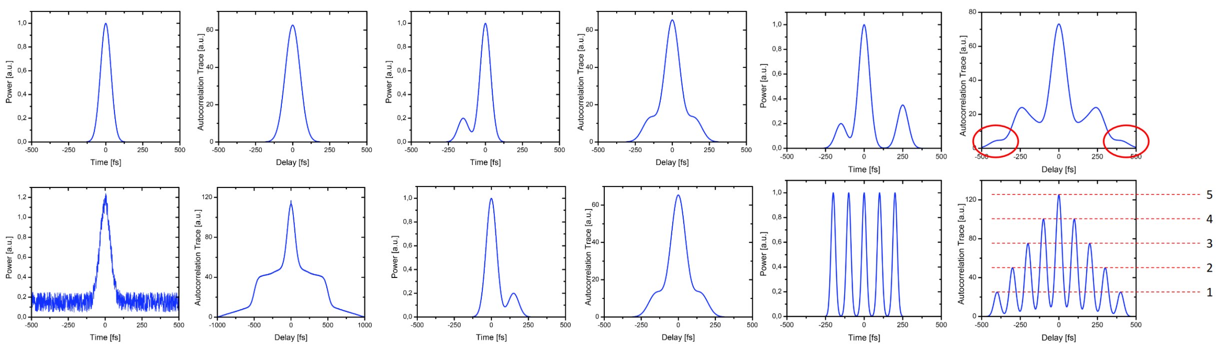


Fig. 5: Examples of autocorrelation functions.